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RELIABILITY-BASED DESIGN AND OPTIMIZATION OF SELF-TWISTING COMPOSITE MARINE ROTORS

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ABSTRACT

The objective of this work is to develop a reliability-based design and optimization methodology to improve the efficiency of self-adapting composite marine rotors. The goal is to quantify the influence of material and operational uncertainties on the performance of self-adapting marine rotors, and to present a design and optimization scheme to maximize the performance and reliability of these structures.

INTRODUCTION

The focus of the current work is on passive, self-adapting marine structures that utilize fluid-structure interactions (FSI) to improve structural performance and function. More specifically, the current work presents a reliability-based design and optimization methodology to improve the energy efficiency of self-adapting composite marine rotors. A previously validated 3-D fluid-structure interaction model is used to determine the performance functions to generate response surfaces. The response surfaces were used to perform a Monte Carlo analysis in an effort to evaluate the global sensitivity to material and load uncertainties, and to optimize the design parameters.

The objective of reliability-based design and optimization is to ensure a level of reliability with respect to structural and operational uncertainties through minimization of unacceptable performance. A recent literature review in this area can be found in [1]. Of the available literature, little work focuses on flexible structures that interact with their environment [2]. State-of-the-art methods in reliability-based design and optimization are presented in [2] with regard to aeroelastic structures, where the authors presented an FSI model for the analysis of a 3-D wing structure and the first-order reliability method is used to evaluate performance sensitivities. Probabilistic design methods have been used to optimize the design of composites in [3; 4]. Response surface techniques were used to analyze parametric sensitivities for a thin-walled composite cylinder in [5]. These methods, however, all focused on aerospace structures. Similar probabilistic-based design methodology is also needed for the optimization of adaptive marine structures.

Self-Adapting Composite Marine Propellers

Marine propellers are traditionally made of nickel-aluminum-bronze (NAB) due to its excellent stiffness, yield strength, and anti-biofouling characteristics. They are designed to be rigid, and the blade geometry is optimized to yield the maximum efficiency at the design flow condition. However, when the ship speed or the shaft rotational frequency moves away from the design values, the blade geometry becomes sub-optimal relative to the changed inflow, leading to a decrease in energy efficiency. This problem can be avoided or minimized by using blades made of carbon fiber reinforced plastics (CFRP). In addition to the well-known higher specific stiffness and higher specific strength of CFRP, the intrinsic deformation coupling behavior of anisotropic composites can be utilized to improve the propeller performance by passive tailoring of the load-induced de-

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formations according to the changing inflow. As demonstrated by recent experimental [6; 7] and numerical [8–16] studies, a properly designed self-adapting composite marine propeller can achieve higher energy efficiency and improved hydrodynamic performance compared to its rigid counterpart when operating at off-design conditions or behind a spatially varying wake.

Objectives

Significant advancements have been made recently related to deterministic design and optimization of self-adapting composite rotors. However, material properties, geometry, boundary constraints, operational conditions, and environmental conditions are all subject to natural or man-made random variations. Hence, the objectives of this work are to (1) quantify the influence of material and load uncertainties on the performance of self-adapting composite marine propellers, and (2) optimize the design to achieve the desired level of reliability in the structural performance.

PROBLEM DEFINITION

Reliability-based design and optimization are common practice for many rigid and/or non-adaptive structural engineering systems. The objective is to ensure the reliability requirements with respect to uncertainties in structural parameters and operating conditions by minimizing the probability of unacceptable performance. Performance and probability of failure calculations are all made while explicitly considering uncertainties in structural properties and loading conditions.

To perform a reliability-based evaluation of the structure, two performance measures are evaluated. First, the probability of unsatisfactory performance is found by defining a limit state function, $g(\mathbf{S},\mathbf{R})$, where $\mathbf{S}$ is a vector of design variables, either deterministic or random, $\mathbf{R}$ is a vector of random variables representing uncertain structural properties and loading conditions. The function $g(\mathbf{S},\mathbf{R})$ can either be implicit (e.g., the outcome of a numerical FSI code), or explicit (e.g., an approximate equation obtained using the response surface methodology described below). The function $g(\mathbf{S},\mathbf{R})$ is also chosen such that $g(\mathbf{S},\mathbf{R}) = 0$ defines a boundary between satisfactory and unsatisfactory performance (with $g(\mathbf{s},\mathbf{r}) < 0$ indicating that the structure has unacceptable performance). The performance state associated with the boundary $g(\mathbf{S},\mathbf{R}) = 0$ is denoted a "limit state." Given this formulation, the optimization problem can be written as

$$\min_{\mathbf{S}} \left[p\left(g_{obj}(\mathbf{S},\mathbf{R}) \leq 0 \right) \right] \tag{1}$$

where $g_{obj}(\mathbf{S},\mathbf{R})$ is the objective function, based on the efficiency (η) of the adaptive composite propeller, which is required to be greater than a minimum target efficiency for all flow conditions, ε_η:

$$g_{obj}(\mathbf{S},\mathbf{R}) = \eta(\mathbf{S},\mathbf{R}) - \varepsilon_\eta \tag{2}$$

subject to two probabilistic limit state functions g_1^{prob} and g_2^{prob}.

$$g_j^{prob} = p_{fj} - p\left(g_{fj}(\mathbf{S},\mathbf{R}) < 0\right) \geq 0; j = 1,2 \tag{3}$$

where the constraint functions g_{fj} are defined

$$g_{f1}(\mathbf{S},\mathbf{R}) = 1 - \frac{P_{ST}(\mathbf{S},\mathbf{R})}{P_{rigid}(J)} \tag{4}$$

$$g_{f2}(\mathbf{S},\mathbf{R}) = \frac{\Delta_{max}}{D} - \frac{\Delta(\mathbf{S},\mathbf{R})}{D} \tag{5}$$

subject to an acceptable probability of failure, $p_f = [p_{f1}\ p_{f2}]^T$. We denote $\eta(\mathbf{S},\mathbf{R})$ as the efficiency of the self-twisting propeller and $P_{ST}(\mathbf{S},\mathbf{R})$ and $P_{rigid}(\mathbf{R})$ as the power demand of the self-twisting and rigid propellers, respectively. Note here that the rigid propeller is a function of the loading condition represented by the advance coefficient J only because the objective is to optimize the design variables for the self-twisting propeller such that it yields equal or better performance compared to the already optimized rigid propeller. Hence, the rigid propeller is only used as a reference to evaluate the performance of the adaptive propeller.

In the application considered here, the vector of random variables is defined $\mathbf{R} = [J, E_1, E_2, G_{12}, \nu_{12}, \nu_{21}]^T$, where $J = V/nD$ is the advance coefficient defining the flow condition representing the operational parameter with propeller advance speed V, rotational frequency n, and diameter D; for the sake of simplicity, the blades are assumed to be made of a single layer of orthotropic lamina with material properties $E_1, E_2, G_{12}, \nu_{12}$, and ν_{21} oriented at angle θ counterclockwise relative to the spanwise direction. Again, for simplicity, the only design variable considered is the fiber orientation angle, $\mathbf{S} = \theta$. By running the propeller fluid-structure interaction analysis model [14; 15] for extreme values of the material parameters $(E_1, E_2, G_{12}, \nu_{12}, \nu_{21})$, it was found that these parameters do not have a notable effect on the system response [17]. For the purposes of this paper it is assumed that variations from the design values in these parameters have a negligible effect on propeller performance. As such, the random variable vector $\mathbf{R}$ can be assumed to only contain the advance coefficient, J.

Equation (4) is used to ensure that the required power for the self-twisting propeller is less than that of the rigid propeller for acceptable performance, which will guarantee that the self-twisting propeller provides higher energy efficiency on average. Further, we define $\frac{\Delta(\mathbf{S},\mathbf{R})}{D}$ as blade tip deflection normalized by the diameter, D, of the propeller, a parameter which is limited by the maximum allowable normalized blade tip deflection, $\frac{\Delta_{max}}{D}$. The blade tip deflection needs to be restrained to limit the possibility of blade strength and stiffness failures. Composite blades made of CFRP can have many possible material failure modes, as well as hydroelastic instability failure modes, most of which can be correlated to the tip deflections. As such, the more easily measured tip deflection provides a standard of safety regarding multiple possible structural stability and integrity characteristics. Hence, Eqn. (5) is used to represent the safety limit, while Eqn. (4) is used to represent the serviceability limit. This limits the optimal design range and the objective function (Eqn. (2)) is used to find the fiber orientation angle that optimizes propeller performance as defined by the overall efficiency of the propeller.

PROBLEM SETUP

The propeller herein is modeled using a single layer for simplicity, but the actual model will have many layers and will be stacked in a sequence such that the load-deformation characteristics will be the same as the effective single layer [18]. The material selected is Hexcel IM7-8552 carbon epoxy composite [19]. The mean-load geometry is based on that of propeller 5474 (Fig. 1), one of the composite propellers manufactured by AIR Fertigung-Technologie GmbH and designed and tested in cooperation with the Naval Surface Warfare Center, Carderock Division (NSWCCD). The model-scale propeller has a diameter of $D = 0.6096\ m$. The design rotational frequency is $n = 780\ rpm$. The design advance coefficient is $J = V/nD = 0.66$. More details of propeller 5474 can be found in [7; 12]. For the possible range of J values in forward operation, the self-twisting propeller is designed to be overpitched in its unloaded configuration. The self-twisting propeller de-pitches due to twisting motion induced by bending deformation caused by the fluid loading, which changes with J. The design requirement is that (1) at $J = J_{design} = 0.66$, the deformed geometry of the self-twisting propeller matches the optimized rigid propeller geometry to achieve equivalent performance between the two propellers, and (2) at $J \neq J_{design}$, the self-twisting propeller should outperform its rigid counterpart. The result is a propeller that is, on average, more energy efficient than its rigid counterpart, requiring overall less power to perform and less variation in power, which reduces strain and load fluctuations, and hence extends the fatigue life of the engine. For details about the design procedure or fluid-structure interaction analysis methodology, readers should refer to [10–12; 14–16; 18; 20].

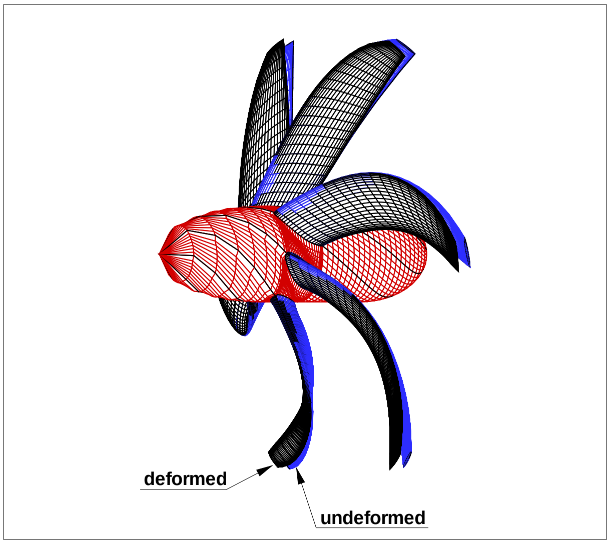

Figure 1. DEFORMED AND UNDEFORMED GEOMETRY OF PROPELLER 5474 AT THE DESIGN FLOW CONDITION.

RESPONSE SURFACE METHODOLOGY

A fully-coupled boundary element method-finite element method (BEM-FEM) model [14; 15] has been developed for the design and analysis of adaptive composite marine rotors. Although the coupled BEM-FEM analysis method is much faster than coupling a Reynolds-averaged Navier-Stokes (RANS) with an FEM method, it can still be computationally expensive to use, with time requirements ranging from 5 minutes to 2 hours for a single simulation, depending on if the analysis is steady, unsteady, with or without cavitation. For a Monte Carlo analysis large enough to successfully achieve a reliable optimization, use of this model becomes impractical. Since the behavior of the performance (power, deflection, and efficiency) are expected to be smooth functions of J and θ, the response surface methodology is a more practical analysis alternative. Specifically ordered data points obtained from the BEM-FEM model were used to predict the behavior of the self-twisting composite propeller which are then used to generate response surfaces via two-dimensional regression analysis. The resulting coefficients of determination, representing the goodness-of-fit of the surfaces, for the power demand, blade tip deflection, and efficiency, respectively, are 0.997, 0.997, and 0.988, where

$$R^2 = 1 - \frac{\Sigma\left(g_{BEM-FEM} - \bar{g}_{BEM-FEM}\right)^2}{\Sigma\left(g_{BEM-FEM} - g\left(\mathbf{S},\mathbf{R}\right)\right)^2} \tag{6}$$

where $g_{BEM-FEM}$ is the data obtained from the BEM-FEM model, $\bar{g}_{BEM-FEM}$ is the mean of all data obtained from the BEM-FEM model, and $g\left(\mathbf{S},\mathbf{R}\right)$ is the data from the regression analysis. Values closer to 1.0 represent higher accuracy. The three response surfaces are shown in Figs. 2,3, and 4. Note that J is dimensionless and θ is in degrees.

The power requirement is more sensitive to J than to θ. Lower values of J correspond to higher angles of attack and higher loads and thereby higher power demands. At higher loads, the change in pitch caused by the fluid-structure interaction is also greater, and hence the power demand is more sensitive to θ at lower J values. At high J values, the power demand is lower and is less sensitive to θ due to small changes in pitch caused by the hydrodynamic load induced bending-twisting deformation.

The maximum deflection is a strong function of both J and θ. This is because, as the fiber orientation angle becomes larger, the blades are less stiff along their primary (longitudinal) axis (which, at $\theta = 45^o$ becomes oriented more as the secondary axis). As a result, the blade tip deflections have nonlinear growth with fiber orientation angle. The increasing of the tip deflection with decreasing J is also expected due to increasing longitudinal load.

The efficiency is highest at the design values ($J = J_{design} = 0.66, \theta = \theta_{design} = 32^o$), which means that the design objectives are satisfied. Note that the efficiency of the adaptive composite propeller has a strong dependence on J, which is inversely proportional to the angle of attack, but a weaker dependence on θ. It is of note, however, that there exists a quadratic element to the behavior of the surface based on the fiber orientation angle. This curvature switches directions at $J = 0.66$ and the local maximum and minimum point is located at $\theta = \theta_{design}$, a characteristic which the response surface takes into account. This change in curvature is because for $\theta > 32^\circ$ and $\theta < 32^\circ$, the change in tip pitch angle, $\Delta\phi$, will be less than $\Delta\phi\ |_{\theta=\theta_{design}=32^\circ}$. For $J < J_{design} = 0.66$, if $\Delta\phi < \Delta\phi\ |_{\theta_{design}}$, the loaded pitch distribution will be further away from the theoretical ideal value and hence $\eta < \eta\ |_{\theta_{design}}$; for $J > J_{design} = 0.66$, if $\Delta\phi < \Delta\phi\ |_{\theta_{design}}$, the loaded pitch distribution will be closer to the theoretical value and hence $\eta > \eta\ |_{\theta_{design}}$.

The rigid propeller power requirement does not require response surface methodology as it is only a function of J; however, fitting a curve to define the behavior of the rigid propeller is also faster than using the BEM model to compute the behavior at each J value (FEM analysis is not needed since the blades are designed to be rigid). Using polynomial fitting techniques, a second-order curve was fit (R^2 = 0.999) to the data for the rigid propeller:

$$P_{rigid}(J) = P_{rigid}(\mathbf{R}) = -38572J^2 - 5091J + 50416 \qquad (7)$$

DESIGN SAMPLE

A design example is presented based on the reliability and response surface methodology above using a standard Monte Carlo analysis. The response surface rather than the fully coupled BEM-FEM propeller FSI solver is used to evaluate the structure performance.

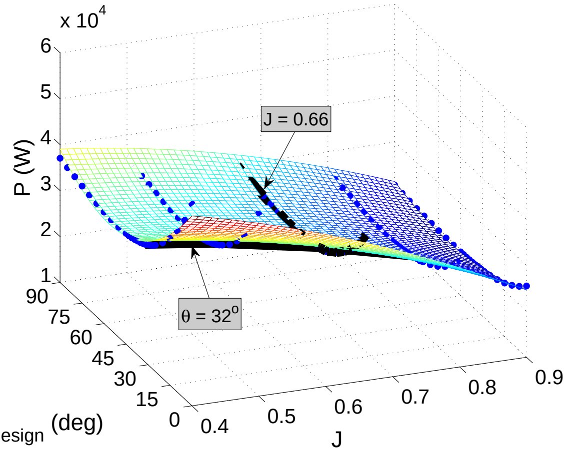

Figure 2. POWER DEMAND, P, RESPONSE SURFACE FOR THE SELF-TWISTING PROPELLER.

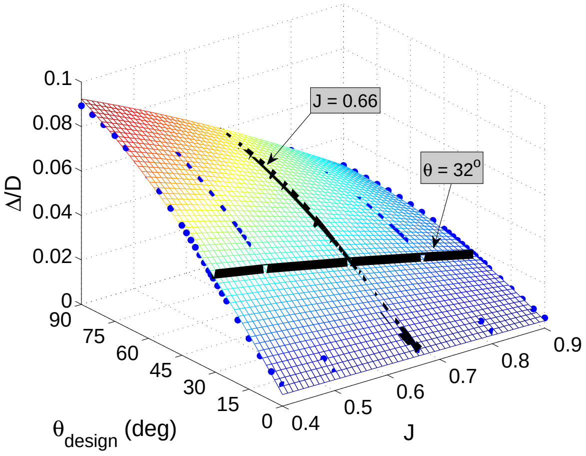

Figure 3. NORMALIZED BLADE TIP DEFLECTION, Δ/D, RESPONSE SURFACE FOR THE SELF-TWISTING PROPELLER.

The first step beyond the response surface methodology involves determining how to define the random distribution of the variables. It is typical for a manufacturer to provide a fiber orientation tolerance around $2-3^o$ in the construction of the laminates for propeller or turbine blades, with a confidence level of 95%. With this as a reference point, it is reasonable to assume that the fiber orientation angle has a Gaussian distribution with a mean value of θ_{design}. A tolerance of 3^o with 95% confidence can be approximated by a standard deviation of 1.5^o (for a normal dis-

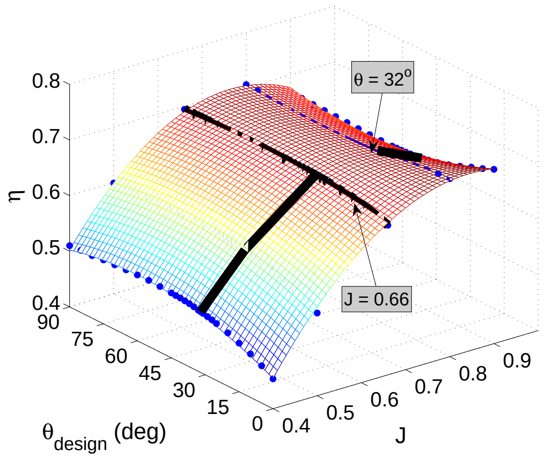

Figure 4. PROPELLER EFFICIENCY, η, RESPONSE SURFACE FOR THE SELF-TWISTING PROPELLER.

tribution, 95% of values are within 2 standard deviations of the mean).

Further, it can be assumed that the propeller will operate near the design advance coefficient ($J_{design} = 0.66$) under most operating conditions and that this would be an appropriate mean value. A standard deviation of 0.10 and a normal distribution will provide a realistic range of operating conditions.

A second step is to define an acceptable maximum tip deflection and minimum target efficiency. An inherent problem of self-twisting propellers is that they can be subject to hydroelastic instabilities and resonance issues. As described above, by limiting the tip deflection these issues can be avoided or minimized. Extending this value too high can lead to static divergence (during deceleration or backing), increased stresses, and higher susceptibility to resonance. The value $\Delta_{max}/D = 0.05$ is selected for this design example, which prevents the stresses from reaching their peak value and sets a minimum allowable natural frequency for the system within a safe range. The minimum target efficiency is set at $\varepsilon_\eta = 0.60$. Finally, the values of p_f must be set. By setting the serviceability limit state to be that the self-twisting propeller outperform its rigid counterpart at least 50% of the time (i.e. $p_{f1} = 0.50$), it is ensured that the self-twisting propeller yields better averaged performance over all possible flow conditions. Further, a balance must be reached between limiting the deflection and allowing the blades to bend and twist enough to provide hydrodynamic efficiency improvements. In this example, $p_{f2} < 0.001$ is used as the constraint to ensure that deflections do not grow to the extent to cause instabilities and excessive stresses.

The results of the objective function and limit states are shown in Fig. 5. According to the top figure, the optimal fiber orientation angle in terms of the objective function is about 59^o; however, there is little variation in the failure probability across the entire range of θ. The limit states play a very important role beyond the objective function. The constraint functions each have definitive boundaries for acceptable performance. The serviceability constraint is that the power requirement of the self-twisting propeller is lower than that of the rigid propeller on average, which can only be satisfied if $31^o \leq \theta \leq 81^o$. Second, the safety constraint limits the fiber orientation angle to $\theta < 34^o$. Hence, what seemed initially to be a wide range of viable options for the design variable based on the objective function is limited to a small range of $31^o \leq \theta \leq 34^o$. In this case, the probability of failure of the objective function ranges between $5.1-5.4\%$, which represents approximately 94% confidence that the self-twisting propeller will exhibit safe and improved performance over the rigid propeller for a realistic range of operating conditions.

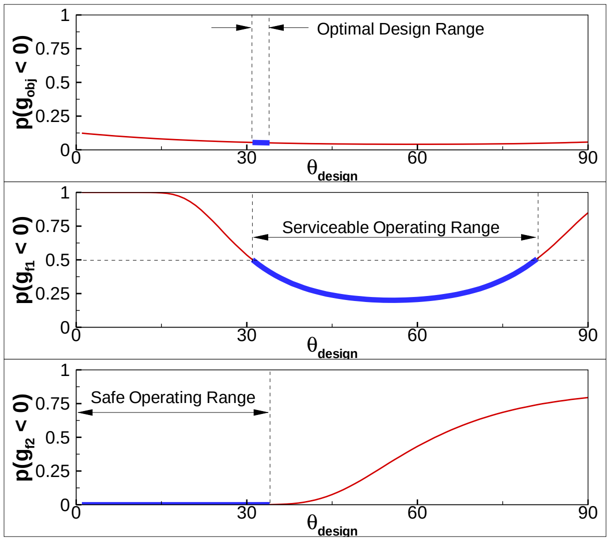

Figure 5. PROBABILITY OF FAILURE OF THE OBJECTIVE AND LIMIT STATE FUNCTIONS.

CONCLUSIONS

The objective of this research is to develop a reliability-based design and optimization methodology to improve the energy efficiency of self-adaptive composite marine rotors while considering material and load uncertainties. Using Response Surface and Monte Carlo analysis, it was shown that the optimal fiber orientation angle for the adaptive propeller is $31^o \leq \theta \leq 34^o$, which will yield a 94% probability of acceptable performance based on three criteria: a serviceability limit state based on propeller power requirement, a safety limit state based tip deflection, and an objective function that ensures a maximum energy efficiency. The serviceability limit state and constraint functions are designed such that, on average, the adaptive propeller outperforms its rigid counterpart. The safety limit state and constraint functions are designed to limit the tip deflection to a specified value to prevent excessive deflections, stresses, and to reduce the susceptibility to hydroelastic instability failures. Finally, the objective function was used to determine the optimal fiber orientation angle that will maximize the energy efficiency of the self-twisting propeller.

The adaptive propeller was previously designed using deterministic techniques to satisfy the following design condition: the adaptive propeller must yield equal or better performance than its counterpart over all flow conditions. The effects of material and load uncertainties were not considered.

In this work, we developed and applied a probabilistic based method to (1) analyze the performance of the adaptive propeller subject to random variations in load and fiber orientation angle, and (2) find the optimal fiber orientation angle that will maximize the energy efficiency while simultaneously minimize the engine power demand and limit the maximum blade tip deformation. The optimal fiber orientation angle ($31^o \leq \theta \leq 34^o$) was found to be similar to the deterministic design ($\theta = 32^o$). However, at $\theta = 32^o$, the probability that the overall energy efficiency of the adaptive propeller will either equal or exceed its rigid counterpart is 94%, which suggests that there are small regions within the design space where the rigid propeller is a better choice than the self-twisting propeller.

The results show that a probabilistic approach is more appropriate than a deterministic approach for the design and optimization of adaptive composite structures that rely on fluid-structure interaction. This is because such structures are inherently more sensitive to random variations in material properties, geometric configurations, and loading conditions. Additional work is needed to assess the effect of material, geometry, and load uncertainties on initiation and evolution of failure modes. This is more complex due to the need to consider the many layers of laminates and the many possible modes of failure, as well as uncertainties in the failure modeling of CFRP.

It should be emphasized here that although the methodologies presented here focused on adaptive composite marine propellers, the framework is also generally applicable to other flexible structures that undergo fluid-structure interactions, including wind or tidal turbines as presented in [21].

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